Genetic Beam Search for Fuzzy Open Shop Problems

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1 Motivation and Problem description

The open shop scheduling problem (OSP) is a problem with an increasing presence in the scheduling literature and with clear applications in industry. It consists in scheduling a set of n jobs to be processed on a set of m machines. Each job consists of m tasks, each requiring the exclusive use of a different machine for its whole processing time without preemption. In total, there are $n \times m$ tasks, denoted $\{o_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$, each one with processing time p_{ij}. A solution to this problem is a *schedule* –an allocation of starting times for all tasks– which is *feasible*, in the sense that all constraints hold, and is also optimal according to some criterion. In this paper, the objective will be minimising the makespan C_{max}, that is, the time lag from the start of the first task until the end of the last one. For a number of machines $m \geq 3$, this problem is NP-complete; in consequence, it is usually tackled via metaheuristic techniques. We highlight an ant colony optimisation hybridised with beam search in [2]; a particle swarm optimisation algorithm proposed in [7] and a hybrid genetic algorithm used in [1].

However, in real-life applications, it is often the case that operation processing times are ill-known, and only some uncertain information is available, for instance, an interval of possible durations or a most likely duration with a certain error. The approaches to deal with this uncertainty are diverse and, among these, fuzzy sets have been used in a wide variety of ways [4]. For instance, uncertain durations are naturally modelled using a *Triangular Fuzzy Number* (TFN) $N = (n^1, n^2, n^3)$ given by an interval $[n^1, n^3]$ of possible values and a modal value n^2 in it.

In the OSP, we essentially need two operations on fuzzy processing times, the sum and the maximum. These are obtained by extending the operations on real numbers using the *Extension Principle.* For any pair of TFNs M and N, $M+N = (m^1+n^1, m^2+n^2, m^3+n^3)$. However, computing the maximum is not that straightforward and the result may not be a TFN. For these reasons, it is usual to approximate the result by a TFN, so $\max(M, N) \approx (\max(m^1, n^1), \max(m^2, n^2), \max(m^3, n^3))$; the approximated and the real maximum have identical support and modal value. The membership function of a fuzzy number can be interpreted as a possibility distribution on the real numbers. This allows to define its expected value, given for a TFN N by $E[N] = \frac{1}{4}(n^1 + 2n^2 + n^3)$. It coincides with several defuzzification indices and induces a total ordering $\leq_E$ in the set of fuzzy intervals: $M \leq_E N$ if and only if $E[M] \leq E[N]$.

Here we consider the fuzzy open shop problem ($FOSP$) with expected makespan minimisation. Some methods have been already proposed for this problem, for instance a particle swarm algorithm in [6] or a lexicographical multiobjective approach in [5]. In the following we propose to hybridise a genetic algorithm with beam search to solve it.

2 Genetic Beam Search

Beam search (BS) is present in the most successful methods for solving the OSP. It was combined with a local search and embedded into an ACO algorithm in [2] and also used as part of the evaluation function of a PSO algorithm in [7]. Roughly speaking, BS are breadth-first search algorithms where the maximum number of nodes per level is limited by a "beam width" (bw). There is also a constant value ext that limits the maximum number of children that a node is allowed to generate. This naive scheme may be enhanced by adding additional features. For instance, an upper bound may be used to prune the tree, thus guiding the search to more promising areas, and a local search can be used at each step, enhancing the exploitation of the algorithm at the cost of increasing the computational effort.

Following [7] our *BS* is guided by an operation priority array. Pruning is achieved by using a formula for the lower bound of partial schedules and an upper bound. We include the search into a genetic algorithm, using it as an evaluation function, which allows to consider different priority arrays and evolve them. In order to have a good tradeoff between the quality of the solutions and the computational cost, we set $ext = 2$ and $bw = 2n$, where n is the number of jobs of the problem.

A critical step in the *BS* is the choice of a good set of candidate operations to expand a node at each iteration. This is not easy in the presence of uncertainty. Here we adapt to the *FOSP* the concept of "conflict set" from the well-known G&T algorithm for the job shop problem. Given a partial schedule, let Ω be the set of all unscheduled operations, and for every $o \in \Omega$ let EST_o denote its earliest possible starting time and $C_o = EST_o + p_o$ its completion time. Then, the minimum completion time is computed as $C^* = \{C_o : E[C_o] \leq E[C_{o'}] \forall o, o' \in \Omega\}$ and the set of candidate operations is given by $\omega = \{o \in \Omega : \exists i \in \{1, 2, 3\}, EST_o^i < C^{*^i}\}$. This set is then reduced with the "ReduceToRelated" method from [2] and using the best solution found so far by the genetic algorithm as upper bound. Finally, the node is expanded with the *ext* tasks having the highest priority.

3 Experimental Results

We test the proposed algorithm on the most challenging instances of the benchmark from [5] for the *FOSP*. These are 170 fuzzy instances built from the $j7$ (size 7×7) and $j8$ (8×8) instances of the well-known benchmark for the *OSP* proposed in [3]. A parametric analysis indicates that the algorithm obtains its best performance using: PMX crossover operator with probability 1, mutation by inversion with probability 0.10, 2/4 tournament between the parents and their offspring as replacement strategy and random mating, ensuring that all individuals are mated. Regarding convergence, 300 iterations are needed for the $j7$ instances and 500 for the $j8$ ones, with average runtimes of 68 and 202 seconds respectively.

We compare the results obtained after 30 runs of our method with those obtained with the lexicographical approach (Lex) in [6] and the PSO algorithm in [5], which are as far as we know, the best for the *FOSP*. The comparison is made in terms of relative errors (*RE*) with respect to the lower bounds specified in [5]. For the $j7$ instances, *RE* values are reduced in average 36% w.r.t. PSO and 34% w.r.t Lex, being these percentages 26% and 24% in the case of the $j8$ instances.

For the sake of completeness, we run our algorithm to solve also the deterministic instances. Even though the algorithm is not specifically designed for crisp problems, it performs quite well on them compared with the state-of-the-art algorithms: the Beam-ACO from [2], the PSO from [7] and the HGAOS from [1]. For the $j7$ instances, we obtain an average *RE* of 2.6%, being 2.5%, 2.3% and 2.7% respectively for the mentioned algorithms. In the case of $j8$ instances, this value is reduced to 2.1% in our approach, and to 2.0%, 1.7% and 2.6% for the other methods. Although it is not the best, our method seems to be quite competitive. Aditionally, it has found a new best solution for the *j8-per10-0* instance with a makespan of 1019.

References

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