Tabu search with diversity control and simulation for an inventory management problem

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1 Introduction and description of the problem

In most inventory management problems, two types of decision have to be taken at the manufacturer level: *when* and *how much* to order to suppliers [2]. It is assumed that setup, carrying and shortage costs are encountered during the year. Usually, inventory management models are characterized by stochastic demand and constant lead times. In contrast, this paper, which generalizes the approach proposed in [3], deals with the situation where there is a constant known demand rate, but probabilistic lead times whose probability distributions change seasonally. Moreover, the lead times for different orders are assumed to be independent, thus crossovers can occur. Therefore, the interactive effects between different cycles (a cycle is defined as the time between two consecutive orders) due to the occurrence of shortages are difficult to model. Consequently, even if the annual *approximated* costs can be analytically computed with a mathematical function f, simulation (of the lead times) is the only way to compute the annual *actual* costs F of a solution. This study is motivated by the management of raw material at a sawmill in North America. Without loss of generality, consider a 52-weeks planning horizon (a time period is a week). A solution (P, S) can be modeled by two vectors P and S defined as follows: $P_t = 1$ if an order occurs at the beginning of period t, and $P_t = 0$ otherwise; S_t is the order-up-to-level of available inventory at the beginning of period t if $P_t = 1$, and $S_t = 0$ if $P_t = 0$. The following reasonable assumptions are made: (A1) it is possible to analytically *approximate* the annual costs with a function $f(P, S)$ relying only on P, S and the probability distributions of the lead times; (A2) it is possible to compute the $F(P, S)$ (i.e. the annual *actual* costs) with a simulation tool; (A3) based on f, it is possible to analytically compute S from P with a so-called $Compute(S \mid P)$ procedure. It means that anytime P is modified, its associated S vector is immediately updated with $Compute(S \mid P)$.

2 Design of a metaheuristic

Due to the non-stationarity in the lead time distribution, the problem is combinatorial in nature (choice of the P_t's and the S_t's). Moreover, simulation is required to compute the actual cost of a solution. Thus, it makes sense to use (meta)heuristics. The reader is supposed to be familiar with the metaheuristic literature and is referred to [1, 4] for more information on it. The solution space $X(N)$ is defined as the set of all the solutions (P, S) with $\sum_{t=1}^{52} P_t = N$. The general approach consists in providing good solutions for different solutions spaces, starting with $U(N)$ orders and ending with $L(N)$ orders, where $U(N) \leq 52$ (resp. $L(N) \geq 1$) is an upper (resp. a lower) bound on N. At the end, the best solution (over all the considered $X(N)$'s) is returned to the user.

For a fixed solution space $X(N)$, the following steps are performed: (S1) generate an initial solution (P, S) with N orders as equi-spaced as possible; (S2) based on f, try to reduce the approximate costs of (P, S) with a tabu search $TS_f(P, S)$ working on P; (S3) based on F and without changing P, apply a descent local search $DLS_F(S \mid P)$ working on S (a move consists in augmenting or reducing one of the S_t's, by one unit). In $TS_f(P, S)$, a move consists in putting an order earlier or later, but without changing the global sequence of orders. At each iteration, the best non tabu move is performed. If an order is moved, then it is forbidden (tabu) to move it again for tab (parameter depending on N) iterations. The stopping condition is a maximum number $Iter$ (parameter) of iterations without improvement of the best visited solution.

An extension of $TS_f(P, S)$, denoted $TS_f^M(P, S)$, is now proposed for step (S2). Instead of only providing a single solution, an idea is to provide a set M containing m (parameter) promising local optima (promising according to the *quality* function f and a *diversity* function $Div(M)$). To achieve this, additional ingredients are now defined. The *distance* between P and P' is defined as $Dist(P, P') = \sum_{t=1}^{52} \mid P_t - P'_t \mid$. The average *distance* between P and a set M of solutions is defined as $Dist(P, M) = \frac{1}{|M|} \sum_{P' \in M} Dist(P, P')$. The *diversity* of a set M of solution is computed

as $Div(M) = \frac{1}{|M|}\sum_{P\in M} Dist(P, M - \{P\})$. M is initialized with solutions randomly generated. Let P be a solution found by tabu search at the end of an iteration. The key idea is the following: P should replace a bad (according to f) solution of M which poorly contributes to its diversity $Div(M)$. More precisely, let M' be the subset of M containing the m' (parameter) worst solutions of M, for which the worst value is f^{**}. Let $P^{(div)}$ be the solution of M' such that $P^{(div)} = arg\min_{P'\in M'} Dist(P', M - \{P'\})$. Then, if $f(P) > f^{**}$, M is not updated. Otherwise, if $Dist(P^{(div)}, M - \{P^{(div)}\}) < Dist(P, M - \{P^{(div)}\})$, then P replaces $P^{(div)}$.
The resulting metaheuristic is summarized in Figure 1. The returned solution is (P^*, S^*) with an actual cost of F^*, which is the best solution visited in all the considered solution spaces.

Algorithm 1 General approach

Initialization: set $F^* = \infty$ and $N = UB(N)$
While $N \geq LB(N)$**, do**
1. generate an initial solution P with N orders as equi-spaced as possible
2. apply $TS_f(P,S)$ or $TS_f^M(P,S)$, and let $M = \{P^{(1)}, \ldots, P^{(m)}\}$ be the resulting set of local optima according to f ($m = 1$ if $TS_f(P,S)$ is used)
3. for $i = 1$ to m, do: apply $DLS_F(S \mid P)$ on $(P^{(i)}, S^{(i)})$
4. set $(P,S) = arg\min_{i\in\{1,\ldots,m\}} F(P^{(i)}, S^{(i)})$
5. if $F(P,S) < F^*$, set $(P^*, S^*) = (P,S)$, and $F^* = F(P,S)$
6. reduce N by one unit

3 Results and conclusion

The experiments were performed on a PC Pentium 4 (1.6 GHz/1 Go RAM). The parameters $Iter$, m and m' were respectively tuned to 1000, 10, 3. As the proposed method has to plan the orders for a whole year, the computing time is not an issue (but all the proposed methods never exceed an hour of computation). Each instance is characterized by its cost parameters (the fixed setup cost A per order, the inventory cost h per unit per period, the shortage cost B per missing unit). For each period t is known the minimum (resp. most likely and maximum) lead time a_t (resp. m_t and b_t). From these three values, discrete triangular distributions can be easily constructed. Two types T_1 and T_2 of instances were generated according to two sets of lead time distributions, with 24 instances per type (which differ according to A, h and B). Set T_1 is based on realistic data from the sawmill context, and is characterized by $a_t \in \{2,5\}$, $m_t \in \{3,7\}$, and $b_t \in \{6,13\}$. Set 2, which represents a form of sensitivity analysis (the variation of the lead times is larger), is characterized by $a_t \in \{1,8\}$, $m_t \in \{2,10\}$, and $b_t \in \{5,16\}$. In Table 1 is provided a summary of the average percentage improvements (over a basic constructive heuristic based on an EOQ analysis) provided by the general proposed approach relying on $DLS_f(P,S)$ (where a descent local search is performed at step (S2) instead of tabu search), $TS_f(P,S)$ and $TS_f^M(P,S)$, respectively. The results are shown for three levels of B and for the two sets T_1 and T_2. Unsurprisingly, the potential benefits of the three methods augments as the seasonality is increased. One can observe that $TS_f^M(P,S)$ outperforms $TS_f(P,S)$, and both methods are better than $DLS_f(P,S)$.

Method	Set T_1			Set T_2		
	Small B	Average B	Large B	Small B	Average B	Large B
$DLS_f(P,S)$	1.39	1.52	1.61	3.75	3.58	3.47
$TS_f(P,S)$	1.72	1.82	2.01	4.15	4.05	3.74
$TS_f^M(P,S)$	1.82	1.86	2.16	4.18	4.06	3.79

Table 1. Compact comparative results

References

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