Metaheuristics in minmax-regret interval data combinatorial optimization problems

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1 Introduction

In combinatorial optimization with interval data, it is assumed that numerical parameters (coefficients) that define the objective function are not known and can take on any values in some pre-specified intervals of uncertainty. Each such possible realization of coefficients is called a scenario, and the set of possible scenarios is the Cartesian product of the uncertainty intervals. In minmax regret combinatorial optimization, the goal is to find a feasible solution that would be ε-optimal for any possible scenario, with ε as small as possible.

In the recent years, much research has been focused on developing efficient computational methods for interval data minmax regret (IDMR) combinatorial optimization problems. IDMR versions of classical combinatorial optimization problems are naturally much more difficult than their classical counterparts, and the challenge is to develop methods that can take advantage of both the structure of the original problem without uncertainty and the structure of the minmax regret objective with the interval data uncertainty set.

2 Contribution

In this talk, we will discuss some computational methods for interval data minmax regret versions of two classical combinatorial optimization problems: the set covering problem (SCP) and the quadratic assignment problem (QAP). The SCP without uncertainty is NP-hard but “computationally friendly”, whereas the QAP without uncertainty is notoriously computationally difficult.

Set Covering Problem. Let $A=(a_{ij})$ be a given 0-1 $m \times n$ matrix. We say that a column j covers a row i if $a_{ij}=1$. A covering is a subset X of columns such that each row is covered by at least one column from X. Assuming that each column j has a weight c_j, the classical covering problem is to find a covering of minimum weight. The IDMR version of the problem is obtained by assuming that the weights c_j are uncertain but belong to pre-specified uncertainty intervals $[c^-_j, c^+_j]$, and considering the minmax regret objective.

Quadratic Assignment Problem. Suppose that there are n facilities that should be assigned to n locations. Let $N=\{1,2,...,n\}$. For any i,j,k,l from N, let $f_{ij}\geq 0$ be the flow from facility i to facility j, and $d_{kl}\geq 0$ be the travel distance from location k to location l. An assignment of facilities to locations can be represented by an $n \times n$ matrix $X=(x_{ij})$, $x_{ij}=1$ if facility i is assigned to location k and $x_{ij}=0$ otherwise. Given an $n \times n$ flow matrix $f=(f_{ij})$ and an assignment $X=(x_{ij})$, the corresponding cost of the assignment is $\sum_i \sum_j \sum_k \sum_l f_{ij} d_{kl} x_{ik} x_{jl}$. For a given flow matrix f, the classical QAP is to find an assignment of minimum cost. The IDMR version of the problem is obtained by assuming that the flows f_{ij} are uncertain and belong to pre-specified uncertainty intervals $[f_{ij}^-, f_{ij}^+]$, and considering the minmax regret objective.

For IDMR versions of both problems, a combination of a metaheuristic with Benders decomposition turned out to be very effective. For the IDMR SCP, a special version of Benders decomposition is used within a hybrid genetic algorithm to improve the crossover operator. For the IDMR QAP, a tabu search heuristic is used within Benders decomposition scheme for the master and slave subproblems.

References

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