A new exact method finding a feasible solution to the bi-level energy pricing problem with two objectives at the lower level

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1 Introduction

Nowadays the optimization of residential loads under varying hourly energy prices is a topic of interest. The embedding of home automation systems intends to help customers to control and manage smartly with their energy consumption. In this work we tackle an energy pricing problem in which an energy provider maximizes its profit that is the total revenue received from selling energy to customers minus total expenses associated with energy peak costs in a given scheduling horizon. At the same time customers minimize their electricity bills and scheduling properly all their appliances. Thus the customers follow the several objectives at the same time which is more realistic situation. We model this problem as a bi-level program with two objectives at a lower level. Bi-level optimization is an appropriate mathematical programming tool if one wants to coordinate a hierarchical organization where a decision at the upper level is influenced by a decision at a lower level [2, 3]. In such case multi-objective optimization should be involved. Here we tackle a bi-level problem with two objectives and the discrete variables at a lower level. Multi-level multi-objective optimization problems are inherently difficult, in particular, because of multiple lower level optima, non-convex feasible region and computational complexity. We propose a new exact method to find a feasible bi-level solution among a potentially large set of compromise lower level solutions.

2 Problem description

An energy supplier, called a leader, is at the upper level and the customers are at the lower level of the hierarchy. Each customer demands to utilize a set of electrical appliances (e.g., dishwasher, chargers, kettle) within a certain time window during the scheduling horizon. The hourly energy prices offered by the energy supplier are varying within the horizon. Due to smart grid applications the customers have the ability to schedule their loads taking advantage of hourly varying prices to minimize their electricity payments [5]. Besides the leader there is a second energy provider, called a competitor. Each hour for each appliance the customers can choose one provider who proposes the less price. In addition, the customers do not want to postpone for a long time the execution of some of their appliances. They minimize the total waiting time across all the appliances. Thus the lower level problem aims to achieve a trade-off between minimizing the electricity payments and minimizing the waiting time for execution all the appliances in response to the hourly prices proposed by two providers. The competitor's energy prices are given. The upper level problem intends to find the optimal leader's prices. The irregular distribution of customers' loads can cause the energy peak that might be very high. The energy peak for the leader is not desirable because of the associated peak costs. Thus the leader maximizes its total profit that is the total revenue from customers' loads served minus the associated energy peak costs.

3 Solution approach

In [1] the problem with one objective function at the lower level has been posed. In this case the lower level problem is polynomially solvable. Here we deal with the bi-level formulation that contains the discrete combinatorial problem with two objectives at the lower level. The decision version of this two objective lower level problem is NP-complete under a fixed upper level solution.

In general multi-level multi-objective combinatorial problems are extremely difficult. To calculate the upper level objective function value it needs a lower level optimal solution that is one of a set of the compromise (or Pareto efficient) lower level solutions. This set might be potentially large. Thus to choose one compromise solution which is the most interesting for the upper level is a challenging problem. It needs either the enumeration of all of the compromise solutions that is time-consuming procedure or developing special exact approaches avoiding the full enumeration.

Given an upper level solution we propose a new iterative procedure to find one lower level compromise solution which is the most interesting for the upper level. The main idea is based on reducing a feasible region of the lower level problem iteratively. The idea of getting iteratively a whole set or a subset of all the efficient solutions was described in [4] for one-level multiple objective integer linear programming problem. Here we develop the similar idea for a bi-level problem. At the first iteration the feasible region is restricted by two extreme compromise lower level solutions. One solution minimizes electricity payments and second one minimizes the waiting time for the execution of all the customers' appliances. Then at each iteration a new Pareto efficient solution with the better upper level objective function value is searched for. For that we formulate an auxiliary mixed integer linear programming problem. The objective function of that problem is to maximize leader's profit under the fixed energy prices subject to the lower level constraints and some cutting constraints defined by the compromise solutions obtained. In worst case the number of constraints may reach the size of the whole Pareto frontier. However, in average the problem becomes infeasible fast and the procedure stops until reaching the whole Pareto frontier. The auxiliary problem is solved by a linear programming optimization solver. Three cases may occur. The first case, the problem is infeasible. The second case, there is a feasible solution but it is not a compromise one. The third case, there is a new feasible compromise solution. The first case means that there is no a compromise solution that is more interested for the leader than one been already found and the procedure stops. In the second case the feasible solution which does not belong to the Pareto frontier might be revealed if it provides us with a better leader's objective value than any compromise solution. In the third case we get a new compromise lower level solution. Then we add a new cutting constraint to the auxiliary problem, start the next iteration solving the problem with the constraints updated.

One of the strategies to find the initial leader's prices is to assign them to the reduced competitor's prices. In such a way the low competitive leader's prices attract the majority or all the customers. Thus the high energy peak and associated costs may appear. Therefore, it becomes more profitable for the leader to increase the price for some hours to get rid of a part of customers' appliances. We use one of the metaheuristics [6], Tabu Search algorithm, to determine these unprofitable hours for the leader in order to decrease the high peak costs and increase the profit.

We implement and discuss simulation results on the randomly generated test instances to evaluate the efficiency of the approach proposed and highlight the further research directions.

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