Stochastic tabu search for the bi–level facility location and design problem

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In the bi–level facility location and design problem, two players (a leader and a follower) compete to attract clients from a given market. Each player has a budget and tries to maximize own market share. At first, the leader opens facilities and determines their design to attract clients. Later on, the follower makes the similar decision for own facilities. Each client splits own demand probabilistically over all facilities in the market proportionally with attraction to each facility and inversely proportional to the distance between client and facility. The location and design of the leader facilities are to be determined so as to maximize his market share [1].

We assume that the set I of potential facility locations and the set J of clients are finite. For each facility $i \in I$ we know the set R_i of design scenarios [2] and this set is finite as well. For each pair $i \in I, r \in R_i$ we have the fixed costs f_{ir} and g_{ir} of opening facility i with design scenario r by the leader and by the follower, respectively. Moreover, we know the attractiveness a_{ir} of the leader facility and the similar parameter b_{ir} of the follower facility. The last two features are important for describing the client behavior. Each client j splits own demand w_j probabilistically over all open facilities directly proportional with attraction to each facility and inversely proportional to the distance d_{ij} between client j and facility i [2]. We consider the utility function u_{ijr} of leader facility i with design scenario r for client j and the similar function v_{ijr} for follower facility:

$$u_{ijr} = a_{ir}/(d_{ij}+1)^{\beta}, \quad v_{ijr} = b_{ir}/(d_{ij}+1)^{\beta}, \qquad i \in I, r \in R_i, j \in J,$$

where β is a distance sensitivity parameter.

Now we introduce the decision variables for the players:
$x_{ir} = 1$ if facility i is opened by the leader with design scenario r and 0 otherwise;
$y_{ir} = 1$ if facility i is opened by the follower with design scenario r and 0 otherwise.

For client j, the total utility U_j from the leader facilities and the total utility V_j from the follower facilities are defined as:

$$U_j = \sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir}, \quad V_j = \sum_{i\in I}\sum_{r\in R_i} v_{ijr}y_{ir}, \qquad j \in J.$$

The total market share of the leader is given by $\sum_{j\in J} w_j U_j/(U_j+V_j)$. The leader wishes to maximize own market share, anticipating that the follower will react to the decision by opening own facilities. The market share of the follower is given by $\sum_{j\in J} w_j V_j/(U_j+V_j)$. The follower maximizes own market share. In opposite to [3], we assume that the players can open facilities at the same site. This Stackelberg game can be presented as the following nonlinear 0–1 bilevel optimization problem:

$$\max_x \sum_{j\in J} w_j \frac{\sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir}}{\sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir} + \sum_{i\in I}\sum_{r\in R_i} v_{ijr}y^*_{ir}} \tag{1}$$

subject to

$$\sum_{i\in I}\sum_{r\in R_i} f_{ir}x_{ir} \le B_l; \tag{2}$$

$$\sum_{r\in R_i} x_{ir} \le 1, \qquad i \in I; \tag{3}$$

$$x_{ir} \in \{0,1\}, \qquad r \in R_i, i \in I; \tag{4}$$

where y^*_{ir} is the optimal solution for the follower problem:

$$\max_y \sum_{j\in J} w_j \frac{\sum_{i\in I}\sum_{r\in R_i} v_{ijr}y_{ir}}{\sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir} + \sum_{i\in I}\sum_{r\in R_i} v_{ijr}y_{ir}} \tag{5}$$

subject to

$$\sum_{i\in I}\sum_{r\in R_i} g_{ir}y_{ir} \le B_f; \tag{6}$$

$$\sum_{r\in R_i} y_{ir} \le 1, \qquad i \in I; \tag{7}$$

$$y_{ir} \in \{0,1\}, \qquad r \in R_i, i \in I. \tag{8}$$

Objective functions (1) and (5) are market shares of the players. Inequalities (2) and (6) are the budget constraints: B_l is the budget of the leader, B_f is the budget of the follower. Inequalities (3) and (7) ensure a unique design scenario for each open facility.

For this Stackelberg game we present stochastic tabu search heuristic based on the optimal and near optimal solutions for the follower problem. For a given solution of the leader, we formulate the follower problem as 0–1 nonlinear multiple–choice knapsack problem and solve it by the branch and bound method. To compute the upper bounds, we relax the integrality constraints and apply the gradient method for this convex problem. Local search approach is used to find high quality feasible solution in initial node of the branching tree. Moreover, we accumulate statistical information by this local search and apply it in new branching rule. The idea of backdoor branching [4] is modified to this end.

In order to find high quality solution for the leader, we apply well–known tabu search framework. New large neighborhood is used in our algorithm. We close one facility and open some other facilities or change design of opened facilities. To reduce the running time of the algorithm we use randomized neighborhood instead of the original one. Starting solution is generated by the alternating matheuristic [1]. Computational experiments for the test instances from [2] indicate that the developed algorithm takes a small number of steps in order to produce optimal or near optimal solutions for this bi–level nonlinear problem.

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