An Island-inspired Genetic Algorithm with Adaptive Parameters Applied to the Multiple Knapsack Problem

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The Multiple Knapsack Problem (MKP) is a well-known NP-hard combinatorial optimisation problem [1] and its goal is to maximize the profit of items chosen to fulfil a set of knapsacks, subjected to constraints of capacity. The problem can be formulated as $max(\sum_{i=1}^{n}(P_i \cdot X_i))$, subject to $\sum_{j}^{m}(W_{ij} \cdot X_i) \leq C_j$ with $X_i \in \{0,1\}$. Where n is number of items, m is number of knapsacks, $P_i \geq 0$ is the profit of item i, $W_{ij} \geq 0$ is the weight of item j, $C_j \geq 0$ is the capacity of knapsack j, and X_i assumes 1 if item i is in the knapsack and 0 otherwise.

Due to its complexity, a large number of heuristics and metaheuristics have been applied to the MKP, especially Genetic Algorithms (GA)[2][3]. GA are inspired by the laws of Darwin where stronger and adapted individuals have greater chances to survive and evolve [4].

This paper proposes an one population island-inspired GA, named *iGA*. Instead of using populations of individuals as islands (island-model GA), our approach uses only one population where each individual is considered to be an island itself. In our proposal, each individual i in the population selects by tournament selection another individual j to migrate. The migration process indicates that individual i will be able to exchange information with individual j. The interaction is made using an uniform crossover that produces one offspring. After that, a bit-flip mutation routine is applied. If the resultant offspring is better than the selected individual (greedy choice), the selected individual is replaced. Otherwise, the offspring is ignored and the selected individual remains the same.

Also, a method to adapt the control parameters (crossover and mutation rates) is applied. It is known that the optimum values of the control parameters can change over the optimization process, directly influencing the efficiency of the method [5]. To obtain the adaptive control of crossover and mutation rates, a set of discrete values is introduced for each of them. A single value is chosen for each parameter at each generation through a roulette wheel selection strategy. The probability of choosing a value is initially defined equally which is subsequently adapted based on a criteria of success. If a selected value for a parameter yielded at least one individual in generation $t+1$ better than the best fitted individual from generation t, then the parameter value has a mark of success. Hence, if at the end of generation $t+1$ the parameter value was successful, its probability is increased with an α value, otherwise, it remains the same. The α is calculated by a linear increase $\alpha = min + (((max - min)/ITER) * i)$, where $ITER$ is the number of iterations, i is the current iteration, max is the maximum value of α and min is the minimum value of α. To ensure a minimum of chance for each value of parameters, a β value is established.

Experiments were run using 11 benchmarks [6]. For each benchmark, 100 independent runs were performed with randomly initialized populations. A simple Genetic Algorithm (*sGA*) was implemented for comparison. It uses tournament selection, uniform crossover and elitism of one individual. Both *sGA* and *iGA* have the same parameters: population size ($POP = 100$), number of iterations ($ITER = 1,000$), tournament size ($T = 3$), 80% of crossover rate, 5% of mutation rate, and elitism of one individual.

The strategy to adapt the crossover and mutation rates is applied in both algorithms, *sGA* and *iGA*, leading to its adaptive versions *A-sGA* and *A-iGA*, respectively. In these cases, the set of values for crossover rate was defined as $\{50, 60, 70, 80, 90\}$ and for mutation rate it was defined as $\{1, 3, 5, 10, 15\}$. A range of $0.01 - 0.1$ was chosen for α and the β parameter was set to 0.01. All of these choices were made empirically.

Table 1 shows the optimum value (V), the number of knapsacks (K) and the number of dimensions (D) for each benchmark. This table also presents the average and the standard deviation of the best result ($Avg \pm Std$) obtained in all runs for each algorithm, the average number of objective function evaluations (*Eval*) required to achieve the optimum value, the success rate (*Success*) calculated as the percentage that the algorithm reached the optimum value, and the dominance information (*Pareto*) indicating which algorithms are better than the others concerning both the average best result and the average number of function evaluations. If more than one algorithm is marked in the same benchmark means that they are non-dominated (neither of them are better than the other in both criteria). Also, for each algorithm, the last line (*Average*) shows the average of evaluations and the average of success rate for all benchmarks.

Analysing the results obtained by *sGA* and *iGA* we can notice that the proposed island-inspired approach obtained much better results (success rate and average of function evaluations) than the *sGA*, except for the instance *PB*5. This gain can be explained by the model used for exchange information that slows down the premature convergence of the algorithm allowing it to better explore the space of solutions.

Comparing the results obtained by *iGA* and *A-iGA*, we can notice that the results (success rate) were even better when using the adaptive parameter control strategy for almost all benchmarks except for instances *PB*5, *PB*7, and *SENTO*1. Also, the average number of function evaluations decreased when using the parameter control strategy. This improvement can be explained by the adaptive choices for the values of parameters during the optimization process.

Analysing the dominance information concerning all algorithms (*Pareto*), it is possible to notice that the proposed approach with adaptive parameter control, *A-iGA*, is present in the non-dominated set in 9 out of 11 instances. This indicates that *A-iGA* is robust concerning both criteria.

Benchmark				sGA				iGA			
	V	K	D	Avg±Std	Eval	Success	Pareto	Avg±Std	Eval	Success	Pareto
PB1	3090	4	27	3085.26±10.78	34995.18	82.00%		3090.00±0.00	13912.02	100.00%	x
PB2	3186	4	34	3131.08±40.44	89051.75	17.00%		3173.19±17.20	74237.64	51.00%	
PB4	95168	2	29	95071.01±551.51	9251.30	97.00%		95168.00±0.00	7231.01	100.00%	x
PB5	2139	10	20	2138.15±3.71	29852.48	95.00%	x	2137.13±5.32	30514.89	89.00%	
PB6	776	30	40	769.57±10.49	51759.22	68.00%		775.86±1.39	12657.32	99.00%	
PB7	1035	30	37	1026.34±6.92	92079.76	17.00%		1034.32±2.22	42747.69	83.00%	x
PET7	16537	5	50	16428.88±47.93	100100.00	0.00%		16529.44±10.97	80221.17	60.00%	
SENTO1	7772	30	60	7640.90±50.75	100100.00	0.00%		7771.64±2.06	47496.00	97.00%	x
SENTO2	8722	30	60	8620.05±37.74	100100.00	0.00%		8717.77±5.71	87966.44	49.00%	
WEING8	624319	2	105	566282.95±12678.93	100100.00	0.00%		612963.36±2750.51	100100.00	0.00%	
WEISH30	11191	5	90	10824.70±92.10	100100.00	0.00%		11159.03±13.71	100100.00	0.00%	
Average					73408.15	34.18%			54289.47	66.18%	

Benchmark				A-sGA				A-iGA			
	V	K	D	Avg±Std	Eval	Sucess	Pareto	Avg±Std	Eval	Sucess	Pareto
PB1	3090	4	27	3086.98±8.17	45491.35	86.00%		3090.00±0.00	17559.92	100.00%	x
PB2	3186	4	34	3142.10±32.96	91786.79	15.00%		3173.47±18.83	72674.13	54.00%	x
PB4	95168	2	29	94956.92±769.63	21115.21	91.00%		95168.00±0.00	8102.60	100.00%	x
PB5	2139	10	20	2136.62±5.90	33728.52	86.00%		2136.79±5.72	34976.06	87.00%	
PB6	776	30	40	770.64±10.04	46877.06	72.00%		775.89±1.09	12355.48	99.00%	x
PB7	1035	30	37	1024.34±7.98	92400.93	12.00%		1034.12±2.62	43877.91	78.00%	
PET7	16537	5	50	16451.34±50.91	98634.27	6.00%		16530.22±10.11	76512.13	64.00%	x
SENTO1	7772	30	60	7678.39±80.06	95481.81	14.00%		7770.61±3.98	39808.61	89.00%	x
SENTO2	8722	30	60	8649.13±50.80	99942.68	1.00%		8718.85±4.54	71824.83	55.00%	x
WEING8	624319	2	105	583830.05±20597.21	100100.00	0.00%		623388.14±1432.22	72758.08	65.00%	x
WEISH30	11191	5	90	10962.33±189.93	99851.97	3.00%		11190.72±1.02	51125.45	93.00%	x
Average					75037.32	35.09%			45597.74	80.36%	

Table 1. Results obtained.

Overall, it is possible to conclude that the proposed approach is an effective method for optimization and adaptive parameter control. As future research we intend to apply the *A-iGA* in different problem domains.

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