A new memetic approach H2col for graph coloring

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1 Introduction

The Graph Coloring Problem (GCP) is very famous because lots of applied problems can be modeled as a GCP : frequency assignment, timetabling, scheduling, fly level allocation, register allocation, printed circuit testing, Sudoku, map coloring... GCP is NP-hard [1], therefore only problems on simple graphs (generally speaking with less than 100 vertices) can be solved by exact algorithms in a reasonable time.

For more difficult graphs heuristics as used. Local searches such as tabu search [2] or variable space search [3] start from an initial coloring and try to improve it by local moves. The best methods to solve the GCP for large graphs are hybrid algorithms such as memetic algorithms [4] or quantum annealing [5]. Those hybrid algorithms use a powerful local search algorithm inside a population-based algorithm.

We present in this paper, H_2col, a new and very simple hybrid algorithm providing the best results of graph coloring for several DIMACS benchmark.

2 Algorithm : H_2col

Our algorithm is a variation of the Hybrid Evolutionary Algorithm (HEA) of Galinier and Hao [4]. HEA is a memetic Algorithms [6], a specific evolutionary algorithms where all individuals of the population are local minimum; the mutation is replaced by an improvement of the TabuCol of [2].

Our algorithm uses the same space search as HEA and TabuCol: the number of colors is fixed, only complete colorings are taken into account (one color is assigned to all vertices) and colorings with conficting edges are allowed (one edge is in conflict if its vertices have the same color). Then the aim is to find a coloring that minimizes the number of conficting edges (until zero).

HEA is one of the best algorithms for solving the GCP; it provided since 1999 the majority of the best results for DIMACS benchmark graphs, particularly for difficult graphs such as DSJC500.5 and DSJC1000.5. These results were obtained with a population of 10 individuals.

In our work, the population of HEA is reduced to only two individuals. To avoid the similarity of the two individuals, we add rules that reintroduce diversity with old individuals (elite solutions).

The pseudo-code of H_2col is detailed in algorithm 1. We consider the same crossover as in HEA, the Greedy Partition Crossover (GPX) based on color classes. Our few modifications on HEA provide important improvements on the quality of the results.

3 Results and conclusion

H_2col have been test on graphs from the second DIMACS challenge of 1992-1993 [7]. The computational experiments show that H_2col finds very quickly the best existing results such as 47-colorings for DSJC500.5, 82-colorings for DSJC1000.5, 222-colorings for DSJC1000.9 and 81-colorings for flat1000_76_0, which have so far only been found by quantum annealing [5] with a massive multi-CPU.

Algorithm 1 H_2col : Hybrid approach with 2 trajectories-based Optimization for graph coloring

Input: $nb\_cycle = 10$.
Output: the best configuration found
$p_1, p_2, elite_1, elite_2 \leftarrow$ init() {initialize with random colorings}
$currentLoop \leftarrow 0$
repeat
  $c'_1 \leftarrow$ GPX(p_1, p_2)
  $c'_2 \leftarrow$ GPX(p_2, p_1)
  $c_1 \leftarrow$ TabuCol(c'_1)
  $c_2 \leftarrow$ TabuCol(c'_2)
  $elite_1 \leftarrow$ saveBest(c_1, c_2, $elite_1$)
  $p_1 \leftarrow c_1$
  $p_2 \leftarrow c_2$
  $best \leftarrow$ saveBest($elite_1, best$)
  if $currentLoop \% nb\_cycle = 0$ **then**
    $p_1 \leftarrow elite_2$
    $elite_2 \leftarrow elite_1$
    $elite_1 \leftarrow$ init()
  end if
  $currentLoop{+}{+}$
until $nbConflicts > 0$ and $p_1 \neq p_2$
return $best$

The second benefit of H_2col is that its computation time is shorter than for others algorithms. We analyze why our slight modifications bring much difference and how manage the correct dose of diversification.

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