Dynamic Combinatorial Auction Problem solved by hybrid metaheuristic based on fuzzy dominance relation

Larbi Asli[1], Talbi El-Ghazali[2], and Méziane Aïder[3]

[1] UMBBoumerdes, Fac. Sciences, Dep. Mathematics, Algeria.
aslilarbi@yahoo.fr
[2] INRIA Lille Nord Europe Parc Scientifique de la Haute Borne 40, avenue Halley Bt.A, Park Plaza
59650 Villeneuve d'Ascq
El-ghazali.Talbi@lifl.fr
[3] LaROMaD, Faculty of Mathematics, U.S.T.H.B., B.P. 32 El Alia, Bab Ezzouar 16111 Algiers, Algeria.
m-aider@usthb.dz

1 Introduction

The Combinatorial Auction Problem (CAP) is defined as a set of articles submitted to sale facing many buyers. For reasons of complementarity between articles, every buyer wants to buy its own subset of articles for which he provides an estimate. Conflicts arise between buyers possible interactions between subsets. The objectives of the seller are different. The most important one is to maximize the total profit of the sale. To achieve the sale that will bring him more he must solve a NP-hard combinatorial optimization problem. All existing Combinatorial Auction models in the literature, do not allow the bidder to renew its offer. This is due to the static mechanisms used. However, in reality, bidders are in competition while it be left time for the auction.

2 Dynamic Bi-objective Combinatorial Auction model

We present here a dynamic bi-objective mathematical model of combinatorial auction. The first objective is to maximize total profit obtained by the auction, and the second one is the liquidation of the maximum number of articles.

The most important parameters of the model are the number of articles m, that will be contained in a vector β, the number of units for each component β_i, and an interval T of time, called "exercise period" during which bidders can launch their bids.

During the exercise period T, bidders launch their bids $S_j(c_j(t), a_{ij}(t))$,where

$c_j(t):$ the value of the offer made by the bidder E_j at time t ($t \in [0, T]$);
$a_{ij}(t):$ the number of units of articles i requested by the bidder E_j at time t;

The sum of the winning bids must not exceed the number of available articles. Let the decision variable $x_j = 1$ if the offer of E_j is accepted, 0 otherwise.

The full model at time $t \in [0, T]$ is:

$$CAP_t = \left| \begin{array}{l} f_t^1(x) = \max \sum_{j=1}^{k} c_j(t) x_j \\ f_t^2(x) = \max \sum_{i=1}^{m} \sum_{j=1}^{k} a_{ij}(t) x_j \\ Sc: \\ \qquad \sum_{j=1}^{k} a_{ij}(t) x_j \leq \beta_i \qquad i = \overline{1..m} \\ \qquad x_j \in \{0,1\}, \qquad\qquad j = \overline{1..k} \end{array} \right. \tag{1}$$

Upon completion of the exercise period $t = T$, the model will be final and contains n bidders.

3 Exploration of the idea and resolution

At the opening of the auction (t=0), bidders launch their bids, the first bidder E_1 launches the bid S_1 at time t_1. It will be represented by a vector of dimension $m+1$ where each component contains the number of units requested by the article a_{1j}. Thus, the value c_1 of the bidder E_l will be considered temporary winner. At time t_2, a second bidder E_2 launches its submission S_2. If the second bid S_2 is not in conflict with the first bid S_1, the bidder E_2 will be a temporary winner. Otherwise, we check if the bidder E_2 dominate the bidder E_1, then E_2 replace E_1 in the list of temporary winner. From this moment we can have: a new bid S_j, or an update of a bid already made. The model is completed until the end of the exercise period $t = T$. Each event will be treated in a manner that will prevent us to better get to the resolution of temporary problem WDP_t.

4 Bi-CAP algorithm with fuzzy dominance relation.

The nature of multi-objective CAP requires us to use fuzzy logic at two levels of resolution. First, when comparing each new solution to already found efficient solutions set (due to its very large size). Second, when choosing a single solution at the end of the resolution process.

Definition 1. *The fuzzification of Pareto dominance relation can be written as follows: It is said that vector a dominate vector b by degree μ_a with*

$$\mu_a(a,b) = \frac{\prod_i \min(a_i, b_i)}{\prod_i a_i} \tag{2}$$

Then we have: if a Pareto-dominating b $\mu_a = 1$, if b Pareto-dominating a $\mu_a = 0$ and $\mu_a \in [0,1]$ otherwise.

Algorithm 1 FM-CAP algorithm

```
while t < T do
  generate a new event;
  if event=new submission then
    size(CAP_t) = size(CAP_{t-1}) + 1;
    if no conflict then
      update set of temporary winners;
    else
      resolve CAP_t by Fuzzy Hybrid Branch&Bound and Tabu Search algorithm;
    end if
  end if
  if event= update submission then
    if conflict between submissions then
      resolve CAP_t by Fuzzy Hybrid Branch&Bound and Tabu Search algorithm;
    else
      update set of temporary winners;
    end if
  end if
end while
```

References

1. Mario Köppen, Raul Vicente-Garcia, and Bertram Nickolay, Fuzzy-Pareto-Dominance and its Application in Evolutionary Multi-Objective Optimization; Fraunhofer IPK,Pascalstr. 8-9, 10587 Berlin, Germany.
2. Tobias Buer, Herbert Kopfer: A Pareto-metaheuristic for a bi-objective winner determination problem in a combinatorial reverse auction; Chair of Logistics, University of Bremen, P.O. Box 330440, 28334 Bremen, Germany.