# A posteriori Pareto front diversification using a Copula-based Estimation of Distribution algorithm

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## 1 Introduction

In most of Multiobjective optimization methods, the decision maker has to run the algorithm to get the optimal choices (Pareto front), and if the obtained solutions have to be updated, the decision maker must *Run the algorithm again waiting for expected new solutions*. This re-execution may cause a lose of time and memory space, so the complexity is increased. To overcome this problem, we propose a new approach known as a Copula-based Estimation of Distribution Algorithm, that allows dynamic solution updates with very low complexity.

An Estimation of Distribution Algorithm (EDA) is a class of the evolutionary algorithms that aims to estimate a distribution of a set of solutions usually the best ones, and use this estimation to generate new ones in every generation. The main difference between an EDA to another optimization algorithm is the manner of the estimation and the fashion of the algorithm implementation.

In Mathematics, a Copula is used to describe the dependencies between random variables. The proposed Copula-based EDA helps to create the estimator of the EDA. After finding the optimal solutions - like any classical optimization Algorithm - the generated Copula Model can be used in a very short time way to find new solutions. To validate our proposal, the proposed algorithm is performed to find the optimal solutions of a set of benchmark problems using the SPEA2 and NSGA2 algorithms as selection methods. After comparison with the original NSGA2 and SPEA2 algorithms, Copula-based EDA gave good solutions in terms of diversity and convergence.

## 2 The Proposed Copula-based Estimation of Distribution Algorithms

The Estimation of Distribution Algorithms uses different ways for estimation, we can find in [1] a good description of the used methods of estimation, however the use of Copula to estimate the distribution in EDA is a very strong idea to optimize complex problems [2] [3] [4]. We referred to copulas as "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions and as distribution functions whose one-dimensional margins are uniform." [5]. Many types of Copula heve been applied in various research studies such as [6] and [7] [8] [9], in this paper, we will use an Archimedeon copula to find the best estimation.

**Algorithm 1** Copula-based EDA

$Q_0 \leftarrow$ Initialization($N_0$)
$NSet_0$ → NDSet$_0 \leftarrow$ Sorting($Q_0$)
$P_0 \leftarrow$ SelectFromNDS(N)
t $\leftarrow$ 1
**while** Not termination criteria **do**
    $C_t \leftarrow$ EstimateMarginal($P_{t-1}$)
    $P_{tmp} \leftarrow$ GenerateSolutions($C_t$)
    NDSet$_t \leftarrow$ Sorting($P_{tmp} \cup$ NDSet$_{t-1}$)
    $P_t \leftarrow$ SelectFromNDS(N)
    NDSet$_t \leftarrow P_t$
    t $\leftarrow$ t+1
**end while**
Return NDSet$_t$,C

**Algorithm 2** Update Solutions

**function** UPDATESOLUTION($C$)
    $P_{tmp} \leftarrow$ GenerateSolutions(C)
    NDSet $\leftarrow$ Sorting($P_{tmp}$)
    P $\leftarrow$ SelectFromNDS(N)
    NDSet $\leftarrow$ P
    Return NDSet
**end function**

Like any evolutionary algorithm, our proposed method has two main steps; the *Selection* and the *Reproduction*, in the first step, we use the *NSGAII* [10] or the *SPEA2* to select the best solution which will be used in the second step called the Reproduction, in this second step, the Copula is applied to estimate, then to regenerate new individuals. The Update function uses the obtained Copula model to diversify the Pareto in a very small time. A pseudo-code of the Estimation of distribution algorithm using a Copula and the solutions' update are illustrated in Algorithms 1,2.

## 3 Experimentation

To proof the efficiency of the proposed algorithm, a set of tests has been conducted using a set of benchmarks which are usually used in this kind of problems trying to test new solving algorithms in the area of Multiobjective optimization. The work of Zitlzer et al. in [11] provided a set of test benchmarks to compare the new algorithms with the classical algorithms citing $NSGAII$ , $SPEA2$. To quantify the quality of our obtained Pareto Solution (PS), the Hypervolume indicator has been chosen as a convergence metric, as well as the $\Delta$ metric which has been considered as a diversity metric. In the other hand, to show the new aspect guaranteed with our proposal which is the ability to get new PS with a very small time (negligible), we have proposed a metric that calculates the number of different PS between two sets of PS.

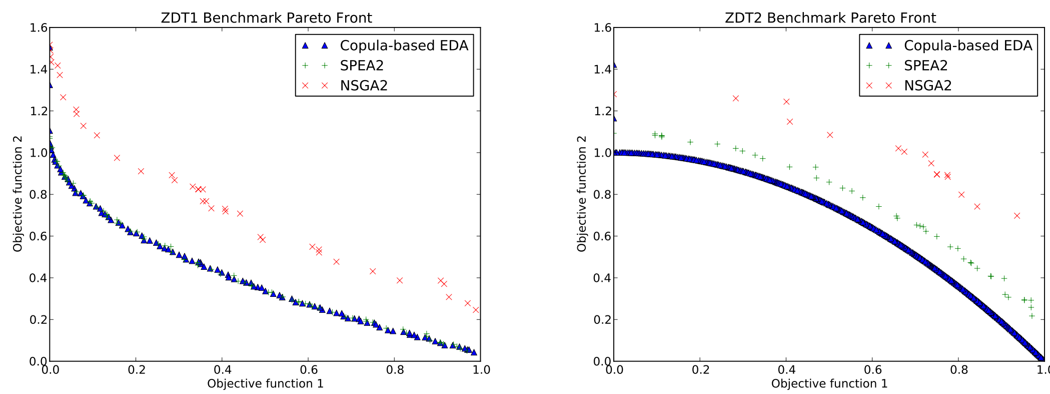


**Fig. 1:** Pareto front of ZDT1 and ZDT2 problem using SPEA2 and NSGA2 respectively as a selection method for the Copula-Based Estimation of distribution algorithm

## 4 Conclusion

In this work, we have proposed three main contributions: 1) the proposal of a new Estimation of distribution Algorithm. The proposed algorithm used a very useful estimation method in statistics which is the Copula and a very famous type which is the Archimedean one and 2) the application of the new Copula-based EDA algorithm to solve Multiobjective Optimization Problems then 3) the use of the obtained Pareto Solutions Estimated Model to generate new Pareto Solutions in a very efficiency manner. The new generated solution may enrich the Decision's choices space of a Decision Maker. The Copula Model can be viewed as a memory that conserves the characteristics of the PS, this vision has given us the motivation to use this algorithm in the Dynamic Multiobjective algorithm, this work can be cited as a future study which uses the Copula Model as a memory, It is worth mentioning that the memory-based algorithms are a class of methods used in Dynamic Multiobjective optimization that provide good results.

## References

1. Hauschild, M., Pelikan, M.: An introduction and survey of estimation of distribution algorithms. Swarm and Evolutionary Computation **1** (2011) 111–128

2. Salinas-Gutiérrez, R., Hernández-Aguirre, A., Villa-Diharce, E.R.: Estimation of distribution algorithms based on copula functions. In: Proceedings of the 13th Annual Conference Companion on Genetic and Evolutionary Computation. GECCO '11, New York, NY, USA, ACM (2011) 795–798
3. Wang, L.F., Zeng, J.C.: Estimation of distribution algorithm based on copula theory. In: Exploitation of linkage learning in evolutionary algorithms. Springer (2010) 139–162
4. Salinas-Gutiérrez, R., Hernández-Aguirre, A., Villa-Diharce, E.R.: Using copulas in estimation of distribution algorithms. In: MICAI 2009: Advances in Artificial Intelligence. Springer (2009) 658–668
5. Nelsen, R.B.: An introduction to copulas. Springer (2006)
6. Gao, Y., Peng, L., Li, F., Liu, M., Hu, X.: Eda-based multi-objective optimization using preference order ranking and multivariate gaussian copula. In Guo, C., Hou, Z.G., Zeng, Z., eds.: Advances in Neural Networks ISNN 2013. Volume 7952 of Lecture Notes in Computer Science. Springer Berlin Heidelberg (2013) 341–350
7. Wang, X., Gao, H., Zeng, J.: Estimation of distribution algorithms based on two copula selection methods. Int. J. Comput. Sci. Math. **3** (2012) 317–331
8. Chang, C., Wang, L.: A multi-population parallel estimation of distribution algorithms based on clayton and gumbel copulas. In: Artificial Intelligence and Computational Intelligence. Springer (2011) 634–643
9. Wang, L., Guo, X., Zeng, J., Hong, Y.: Using gumbel copula and empirical marginal distribution in estimation of distribution algorithm. In: Advanced Computational Intelligence (IWACI), 2010 Third International Workshop on, IEEE (2010) 583–587
10. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. Evolutionary Computation, IEEE Transactions on **6** (2002) 182–197
11. Zitzler, E., Deb, K., Thiele, L.: Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary computation **8** (2000) 173–195