A Multilevel WalkSat Algorithm For MAX-SAT

Noureddine Bouhmala*
Department of Technology and Maritime Innovation
Vestfold University College
Norway

Abstract

This paper introduces a multilevel Walksat algorithm for the Maximum Satisfiability Problem The approach suggests looking at the solution of the problem as a multilevel process operating in a coarse-to-fine strategy. This strategy involves recursive coarsening to create a hierarchy of increasingly smaller and coarser versions of the original problem. The reduction phase works by grouping the variables representing the problem into clusters. This phase is repeated until the size of the smallest problem falls below a specified reduction threshold. A solution for the problem at the coarsest level is generated, and then successively projected back onto each of the intermediate levels in reverse order. The solution at each child level is improved before moving to the parent level. Benchmark including various category of test cases are used to compare the effectiveness of the approach against state of arts algorithms.
Keywords: maximum satisfiability problem, walksat , multilevel techniques.

1 The Maximum Satisfiability Problem

Generally, the satisfiability problem (SAT) which known to be NP-complete [1] is defined as follows. Given a propositional formula Φ consisting of a set of N variables usually represented in CNF (Conjunctive Normal Form). In CNF, the formula is represented as a conjunction of clauses written as $\Phi = C_1 \wedge C_2 \wedge C_3. \wedge ....C_M$, with M being the number of clauses. A clause $C_i(x)$ is a disjunction of literals and a literal is a variable or its negation.

The task is to determine whether there exists an assignment of values to the variables so tha the number of satisfied clause is maximized.

2 The Multilevel WalkSAT

The multilevel Walksat algorithm (MLV-WSAT) which operates in two distinct phases is shown in Algorithm The first phase involves a recursive coarsening to produce smaller and smaller problems that are easier to solve than the original one. Let P_0 denotes the original problem composed on N variables. During the first phase a sequence of smaller problems $P_0, P_1, ...P_k$ such that $|P_0| > |P_1| > |P_2|..... > |P_k|$ is constructed. The next level coarser problem P_1 is constructed from P_0 by merging pairs of variables. The merging procedure is computed using a randomized algorithm. The variables are visited in a random order. If a

*noureddine.bouhmala@hive.no

Instances	$\lvert V\rvert$	$\lvert C\rvert$	**WSAT**	**MLV-WSAT**
fpu8-problem.dimacs24.filtered	160232	548848	6328	3043
fpu-fsm1-problem.dimacs15.filtered	160200	548843	6125	3055
i2c-master2.dimacs.filtered	63816	221320	615	161
b14-opt-bug2-vec1-gate-0.dimacs	130328	402707	1763	1279
b15-bug-fourvec-gate-0.dimacs	581064	1712690	7241	3184
b15-bug-onevec-gate-0.dimacs	121836	359040	1122	493
c1-DD-s3-f1-e2-v1-bug-fourvec-gate-0.dimacs	391897	989885	955	60
c1-DD-s3-f1-e2-v1-bug-onevec-gate-0.dimacs	102234	258294	62	2
c2-DD-s3-f1-e2-v1-bug-onevec-gate-0.dimacs	84525	236942	2537	1362
c3-DD-s3-f1-e1-v1-bug-fourvec-gate-0.dimacs	33540	86944	4	4
c3-DD-s3-f1-e1-v1-bug-onevec-gate-0.dimacs	8358	21736	1	1
c4-DD-s3-f1-e1-v1-bug-gate-0.dimacs	797728	2011216	3761	1129
c4-DD-s3-f1-e2-v1-bug-fourvec-gate-0.dimacs	448465	1130672	1834	516
c4-DD-s3-f1-e2-v1-bug-onevec-gate-0.dimacs	131548	331754	439	105

Table 1: SAT2013 Industrial benchmarks: comparison between MLV-WSAT and WSAT

variable x_i has not been matched yet, then a randomly unmatched variable x_j is selected, and a cluster consisting of the two variable x_i and x_j is created. This process recursively iterate until the size of coarsest problem is reached. A random solution is computed at coarsest level consists in assigning a random state (true or false) to each cluster and all the variables assigned to that cluster will get the same state. The second phase starts by using WalkSat from the coarsestr level to the finest level. Every time WalkSat reaches convergence at a given level P_k, the assignment reached at this level is projected back to a parent level P_{k-1} where a new round of Walksat is performed.

3 Some Experimental Results

The performance of MLV-WSAT is compared against WSAT using a set of benchmark instances from different categories (Industrial, Random 2-SAT, Random 3-SAT). This set is taken from the eight Max-SAT 2013 organized as an affiliated event of the 16th International Conference on Theory and Applications of Satisfiability Testing (SAT-2013). Tables 1 compares MLV-WSAT and WSAT using industrial instances. The first and second column show the number of variables and clauses respectively. The results shows a clear superiority of MLV-WSAT compared to WSAT. The multilevel variant gives the best results in 39 cases out 44 cases and similar performance were observed in the remaining 5 cases. The difference in the quality ranges between 33% and 99%.

References

[1] S.A. Cook (1971). The complexity of theorem-proving procedures. Proceedings of the Third ACM Symposium on Theory of Computing, 151-158.