Solving Global Optimization Problems With Constraints Via Reducing Transformation

M. Rahal, D. Guettal and A. Ziadi

Department of Mathematics, University Ferhat Abbas-Setif 1, Algeria
mrahal_dz@yahoo.fr, guettald@yahoo.fr, ziadiaek@yahoo.fr

1 Introduction

The reducing transformation has been essentially developed for global optimization, but it can be used in more general situations, for instance for approximating several variables functions, for solving systems of nonlinear functional equations and for other multidimensional problems see Cherruault [1, 2]. The basic idea is to use a reducing transformation allowing to convert multidimensional global optimization problems into problems of the same type, but depending on a single variable. Then we have at our disposal the known performing methods and techniques available in the one-dimensional case.

A lot of algorithms have been elaborated from implementing this method and they all have proved their efficacy in various situations see Ziadi and Cherruault [7, 8], Ziadi et al [10] and Ziadi et al [11].

In this work, the reducing transformation will be applied to global optimization problems with constraints. This method consists in approximating the objective function f depending on several variables, by a function f^* of a single variable. Then, applying a one-dimensional covering algorithm to f^* see Piyavskii [4], we can obtain an approximation of the global minimum of f. This technique has allowed to solve global optimization problems, Horst and Tuy [3], Törn and Zilinska [6] in which the objective and constraint functions satisfy the Lipschitz condition.

In this paper, we give some results about the reducing transformation method for global optimization problems with constraints which are still very difficult to solve. This method use numerical approximation of α-dense curves to reduce the original Lipschitz multidimensional problem to a univariate one of the same type. Thus, we can solve the problem by using algorithms for minimizing functions in one dimension. We shall give several results concerning the generation of α-dense curves [5, 9] in some kinds of compacts of $\mathbb{R}^n$. Examples of construction of this curves and numerical experiments on several test functions are given.

This paper has the following structure. In Section 2 briefly present the reducing transformation method for global optimization. Section 3 presents several results concerning the generation of α-dense curves in some kinds of compacts of $\mathbb{R}^n$, and examples of construction of this curves. Section 4 presents results of some numerical experiments on test functions taken from literature. Finally, Section 5 concludes the paper.

Keywords: Global Optimization, α-dense curves, reducing transformation, Piyavskii's algorithm, Constrained Optimization.

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