Optimization Algorithms for Multi-objective Combinatorial Problems under Uncertainty

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1 Introduction

Most of real-world optimization problems are multi-objective in nature as they usually involve the simultaneous satisfaction of multiple conflicting objectives. These multi-objective problems arise across many practical applications in diverse areas and are considered as one of the most important and widely discussed research topics. A variety of resolution methods and heuristic approaches to solve such problems have already been deployed [1]. Yet, despite their importance, there is no consideration of uncertainty in the classical multi-objective concepts and techniques which makes their application to real-life problems impossible.

Moreover, uncertainty characterizes almost all practical applications in which the big amount of data provides certainly some imperfections caused by many sources such as missing information, forecasting, data approximation or noise in measurements. These imperfections (or uncertainties) are very difficult to avoid in practice and should be taken into account within the optimization process. However, the classical way to deal with uncertainty is the probabilistic reasoning [2]. Nevertheless, this reasoning is only appropriate when all numerical data are available, which is not always the case. Indeed, there are some situations such as the qualitative aspects and the case of total ignorance, which are not well handled and which can make such a reasoning unsound. To this end, a panoply of non-classical tools for handling uncertainty appears such as fuzzy sets which provide a simple and robust way to express uncertain data [3].

In recent years, the combination of both lines of research: Multi-objective combinatorial optimization and Uncertain optimization, emerged and gained importance since it closely reflects the reality of many real-world problems. However, most of existing approaches for dealing with multi-objective optimization under uncertainty have been often limited to treat the problem as mono-objective by considering the set of objectives as if there is only one [7]. Some other approaches have been focused on treating it in its multi-objective context but with ignoration of uncertainty propagation to the objective functions by using statistical properties like the expectation value [8][9]. Only few works have been proposed to handle the problem as-is without erasing any of its multi-objective or uncertain characteristics by adapting the classical multi-objective methods to interval context [4] [6]. All these remarks lead us to propose a new optimizer for handling multi-objective problems under uncertainty, in which uncertain data are expressed by means of triangular fuzzy numbers and while considering the uncertainty propagation to the set of objective functions to be optimized.

2 Contribution

The aim of our study is to deal with multi-objective problems with fuzzy data, consequently with triangular-valued objectives. Our main idea is to first propose a new Pareto approach for ranking the generated triangular-valued functions [5], since clearly the classical Pareto dominance cannot be used in our uncertain context. At the second stage, we propose a fuzzy extension of two well-known multi-objective evolutionary algorithms: SPEA2 [10] and NSGAII [11] in order to enable them working in uncertainty space, by integrating the proposed Pareto dominance in their fitness assignment strategy and adapting their classical techniques of diversity preservation (i.e density estimation) and archiving to our triangular fuzzy context. The extended algorithms, implemented with the multi-objective module of version ParadisEO-2.0 under linux [14], are subsequently applied to solve a multi-objective variant of vehicle routing problem (VRP) with uncertain demands [12]. In order to evaluate the quality of solutions found by the proposed algorithms, an experimental study based on multi-objective quality indicators such as Hypervolume metric [15], was finally carried out across a set of fuzzy benchmark instances generated at random from the crisp Solomon's benchmark [13] considered as a basic reference for the evaluation of several VRP resolution methods.

As a future work, we intend to refine the algorithmic features by introducing a new fuzzy distance for the density estimation techniques (i.e. crowding distance and nearest neighbor techniques) and to extend the proposed Pareto dominance for ranking other fuzzy shapes like trapezoidal fuzzy numbers. Another perspective will be the extension of multi-objective performance indicators to uncertain context.

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