Branch and Bound algorithm for the two-machine flowshop scheduling problem with availability constraints

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1 Introduction and related works

Scheduling decisions are crucial in manufacturing industries. Indeed, machines should be operated in an efficient way to produce several types of products simultaneously. Thus, an optimal production planning can satisfy the clients' demands. However, machines could be unavailable in the scheduling period for different reasons. Sudden stops of the production could yield a high production loss and could cause a delay in delivery to customers. To keep the manufacturing facilities in good condition, preventive maintenance is planned. Since production scheduling and preventive maintenance planning are the most common problems faced by the manufacturing industry. For example, in the aerospace industry, a preventive maintenance is performed after a fixed number of operations to ensure products' high quality. The scheduling under availability constraints can be classified into two categories : deterministic and stochastic. In the deterministic case, the starting time of the preventive maintenance action is known and fixed in advance. However, in the stochastic case, the starting time of the preventive maintenance is determined in conjunction with the production schedule.

Few Studies have been carried out on the optimization of production scheduling with availability constraint. Their considerations are either modest nor realistic enough. A more comprehensive and realistic model is proposed in this paper. A review on scheduling problems with limited machine's availability has been done in [5].

Taking availability constraints into consideration, the scheduling of jobs in a flowshop composed of two machines in series where the first machine is unavailable for processing is investigated in this paper. The purpose is to find a schedule that minimizes the makespan. The problem is treated in the deterministic case.

A set of N independent jobs has to be performed on flowshop composed of two machines A and B. Each job consists of two operations. The first operation of each job is processed on machine A and the second on machine B. Machines can perform only one job at a time. Machine A is unavailable during a period of previously fixed time due to a preventive maintenance. Preemption of the operations of each job is not allowed. A job is supposed to restart if it is interrupted by the unavailability period.

A similar two machine flowshop problem was already studied in [3] and [1]. Both previous contributions proposed a dynamic programming algorithm as exact method to solve the problem. The proposed methods are too cumbersome due to their dependency on the total processing time of jobs and they take too long to solve a realistic scheduling problem.

The model suggested in this paper differs in the starting time of the unavailability period from [4]. As we consider that the starting time of the unavailability period for the machine A is previously fixed rather than a date that depends on the number of finished jobs.

2 Approach

The problem is proved by [3] to be NP-hard in the lower sense. We propose four tight lower bounds and two global lower bounds. As upper bounds, we use the johnson algorithm proposed by [2], the modified johnson algorithm presented by [1] and a new efficient algorithm that we developed. A Branch and Bound (B&B) algorithm that incorporates: the upper bounds, the lower bounds and the global lower bounds, is proposed to solve the problem.

3 Results and discussion

Computational experiments are conducted on a variety of randomly generated instance to evaluate the performance of the proposed method. Based on the starting time of the unavailability period, the generated instances are subdivided into three groups : at the beginning (Group 1), in the middle (Group 2) and at the end of the time window (Group 3). The Solver is configured to stop if the optimum is found or the time limit of 1 hour is reached. Results are summarized in Table 1. We evaluate the results for each group using the following metrics : the percentage of optimal solutions, the computation time and the gap between the lower and the upper bound.

Table 1. Computational results for the branch-and-bound method

	Group 1			Group 2			Group 3		
N	Optimal Sol (%)	Time (s)	Gap (%)	Optimal Sol (%)	Time (s)	Gap (%)	Optimal Sol (%)	Time (s)	Gap (%)
10	100	0.00	2.11	100	0.00	3.68	100	0.00	3.75
20	100	0.14	0.87	100	0.66	1.86	100	0.06	2.48
30	100	0.04	0.85	100	0.11	0.64	100	0.05	0.64
40	100	401.78	0.22	80	720.22	1.25	100	0.41	1.37
50	90	363.03	0.50	90	366.67	0.26	100	1.49	0.22
60	100	150.86	0.12	100	210.79	0.30	100	0.70	0.25
70	80	815.55	0.13	80	909.35	0.06	100	4.80	0.17
80	70	1080.00	0.07	60	1529.44	0.20	80	738.16	0.15
90	70	1288.18	0.03	50	1997.12	0.04	100	61.84	0.10
100	40	2160.00	0.00	40	2160.00	0.03	60	1769.95	0.06

The experimental results show that the B&B method is effective to find optimal solution for problems with up to 30 jobs in a reasonable amount of computation time. Otherwise, the gap between the lower and the upper bound is less than 4%.

It has been shown that there is an impact of the unavailabilities starting time. We have also noticed that instances where the unavailability period starts at the middle of the time window are the hardest to solve. However, the instances where the unavailability period is fixed at the end of the time window are the easiest to solve.

4 Conclusion

Two things contribute to the success of the proposed method. The first one is the new bounding scheme which computes fairly tight lower and upper bounds. The second one is the ability of the branching rule to take important decisions at higher levels of the B&B search tree. Thus, our solution which is original, gives us accurate results with maximal time reduction.

References

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