A reliable hybrid solver for nonconvex optimization

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1 Introduction

Nonconvex and highly multimodal optimization problems represent a challenge both for stochastic and deterministic global optimization methods. The former (metaheuristics) usually achieve satisfactory solutions but cannot guarantee global optimality, while the latter (generally based on a spatial branch and bound scheme [1], an exhaustive and non-uniform partitioning method) may struggle to converge toward a global minimum within reasonable time. The partitioning process is exponential in the number of variables, which prevents the resolution of large instances. The performances of the solvers even dramatically deteriorate when using reliable techniques, namely techniques that cope with rounding errors.

In this paper, we present a fully reliable hybrid algorithm named Charibde (Cooperative Hybrid Algorithm using Reliable Interval-Based methods and Differential Evolution) [2] that reconciles stochastic and deterministic techniques. An Evolutionary Algorithm (EA) cooperates with interval-based techniques to accelerate convergence toward the global minimum and *prove the optimality* of the solution with user-defined precision. Charibde may be used to solve continuous, nonconvex, constrained or bound-constrained problems involving factorable functions.

2 Cooperation of stochastic and deterministic methods

Global optimization based on Interval Analysis [3] (IA) has considerably expanded during the last decades. IA is a numerical technique that computes a conservative enclosure of a real-valued function over a subdomain. Each quantity is represented by a compact interval with floating-point bounds, and the elementary operations of a numerical computation are carried out using outward rounding. It provides a prominent tool to compute a reliable lower bound of the objective function over a subdomain in a branch and bound algorithm. By keeping track of the best found upper bound $\tilde{f}$ of the global minimum, subdomains that cannot contain a global minimizer are safely discarded. The exploration is stopped when the desired precision ε is reached. Interval branch and bound algorithms have thereafter been endowed with constraint propagation techniques [4] that narrow the bounds of the variables without losing solutions. These techniques are known as Interval branch and contract algorithms (IB&C).

Computing a good solution of the problem (an upper bound of the global minimum) is crucial in branch and bound algorithms to discard suboptimal subdomains of the search-space. A usual technique consists in performing local optimization on each subdomain to improve the best found solution, but this requires a feasible starting point. EA are particularly suited for finding feasible points and satisfactory solutions. Our hybrid solver Charibde is based on a cooperative scheme in which two independent threads (an EA and an IB&C) exchange information through shared memory: i) whenever the EA improves $\tilde{f}$, the evaluation of the best individual is sent to the IB&C thread. $\tilde{f}$ is updated, which leads to a more efficient pruning of the search-space; ii) whenever a punctual evaluation in a subspace improves $\tilde{f}$, the corresponding point replaces the worst individual of the EA thread, thus preventing premature convergence toward local minima.

Charibde also embeds advanced features based on the cooperation of the two threads. A geometrical heuristic exploits the location of the best known solution to exhaustively explore the search-space. It was devised based on the observation that the tedious task of exploring the neighborhood of the global minimizer is more efficient when the best possible $\tilde{f}$ is available. Preliminary results (see Section 3) suggest that it performs better than classical heuristics (depth first, largest first, most promising first). The EA population may also be restarted to introduce diversity. Unlike other approaches, Charibde uses the remaining subdomains in the IB&C to generate the new EA initial population.

3 Numerically certified optimal results

Charibde achieved a numerical proof of optimality for several difficult problems that were up to now deemed unsolvable. Contrary to standard test functions that have a global minimum 0 at $(0, \dots, 0)$ (Griewank function) or have a global minimizer with n identical components (Schwefel function), we have selected five functions with nontrivial global minima: Michalewicz, Egg Holder, Rana [5], Keane [6] and Lennard-Jones clusters. We present *numerically certified* optimal solutions, as well as a comparison with stochastic and deterministic state-of-the-art solvers.

As an application of constrained optimization, we address the problem of solving conflicts between aircraft, that is guaranteeing the separation between aircraft at any time during the cruise phase while minimizing the delay. This is a very combinatorial problem that has never been solved with exact methods under realistic hypotheses. Possible maneuvers include heading changes and allocation of different flight levels (altitudes). We focus on lateral maneuvers, during which aircraft leave and return to their initial routes with a deviation angle. We display standard conflict situations and their optimal solutions numerically proved by Charibde.

References

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