Solution procedures for the generalized discrete (*r*|*p*)-centroid problem

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1 Introduction

In this work we consider a discrete competitive location model which is called (r|p)-centroid problem, leader-follower problem or Stackelberg problem in locations. The model represents a situation where two players, the leader and the follower make decisions sequentially in order to reach certain objectives. Leader and follower want to determine the locations for p and r facilities respectively. The objective of the follower, who makes the decision once the leader has selected its locations, is to maximize the demand captured by its facilities. The objective of the leader is to minimize the maximum demand that the follower could capture, as demand is assumed to be essential, this objective is equivalent to maximize the demand captured by its facilities. We study a generalized model in which the customer's choice rule is defined using a non-increasing capture function, the demand captured by the players is given by the value of this function for the difference between the distance from the demand point to the follower and the distance from the demand point to the leader. Given the set of p locations for the leader, X_p, the solution for the follower is an (r|X_p)-medianod. The solution for the leader is an (r|p)-centroid. We present a linear programming formulation for the generalized (r|p)-centroid problem and analyse some exact and heuristic solution procedures. We show some examples and results considering different capture functions.

2 Statement of the model and linear formulations

Let V be a set of points and C, L be subsets of V with cardinality $|C|=m$ and $|L|=n$. C is the set of demand points, or clients, and L is the set of potential locations for facilities. Let $d_{ij} = d(c_i,l_j)$ be the distance between demand point c_i and potential location l_j. For any c_i in C and any subset X of L, $d_i(X)$ denotes the distance between c_i and X. Every point c_i in C has a weight w_i which represents the demand at point c_i. We consider a market of essential goods, which means that the demand is totally satisfied, that is, the sum of demands served by the firms operating in the market is equal to the total existing demand. The leader wants to open p facilities and the follower plans to enter the market with r facilities, both players want to determine the locations that allows them to reach certain objectives. The objective of the leader is to minimize the maximum demand that the follower could capture. Since demand is essential, this objective is equivalent to maximize the demand captured by the leader. The objective of the follower is to maximize the demand captured by its facilities.

Given the sets X and Y of locations for the leader's facilities and follower's facilities respectively, the demand captured by the follower is $W = \sum_{i=1}^{n} w_i f_i(d_i)$ where $d_i = d_i(Y) - d_i(X)$ and f_i is a non-increasing real function with $0 \le f_i(d) \le 1$, for any demand point c_i and $d \ge 0$. If $\{Y_j\}_{j \in J}$ is the set of feasible solutions for the follower, the problem of the leader or *(r|p)*-centroid problem can be formulated as follows:

$$\min W$$
$$\sum_{j=1}^{n} x_j = p$$
$$\sum_{i=1}^{m} \sum_{j=1}^{n} h_{ij}^k u_{ij} \le W \quad k \in \left[1..\binom{n}{r}\right]$$
$$\sum_{j=1}^{m} u_{ij} = 1 \quad i \in [1..m]$$
$$u_{ij} \le x_j \quad i \in [1..m],\ j \in [1..n]$$
$$u_{ij}, x_j \in \{0,1\}, \quad i \in [1..m],\ j \in [1..n],$$

where $x_j=1$ if the leader opens a facility at point l_j and $x_j=0$ otherwise, $u_{ij}=1$ if clients at c_i use a leader facility at point l_j and $u_{ij}=0$ otherwise, and $h_{ij}^k = w_i f_i(d_i(Y_k) - d_{ij})$. Constraint $u_{ij}=0,1$, may be relaxed and replaced by $u_{ij} \ge 0$. The amount h_{ij}^k is the demand at c_j captured by the follower with the solution Y_k when the leader has a facility opened at l_j and l_j is the closest leader's facility to that demand point.

3 Solution procedures for the generalized *(r|p)*-centroid problem

Some exact and heuristic procedures proposed to solve the binary (r|p)-centroid problem (Alekseeva et al, Biesinger et al., Campos-Rodríguez et al., Roboredo and Pessoa) can be adapted to solve the generalized version. Considering different capture functions (binary, piecewise linear, piecewise concave and piecewise convex), we consider exact and heuristics solution approaches to solve the bi-level location problem. Some results are presented.

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