# A Genetic Algorithm for the Robust Vehicle Routing Problem with discrete scenarios

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## 1 Introduction

The Vehicle Routing Problem (VRP) is a NP-hard problem which aims at defining routes for a fleet of vehicles, such that each vehicle starts and ends its route at the depot node, each customer is visited once, and the vehicle capacity is ensured. A new issue on solving VRP problems can be affected by uncertain parameters, as a consequence of practical issues such as traffic jams, and punctual and unexpected disruptions (*e.g.* road accidents). This work considers the Robust Vehicle Routing Problem (RVRP) in which uncertainty is considered on the travel times/travel cost, defined as a set of scenarios. The RVRP is defined as follows: let $G = (V, A)$ be a connected and directed graph with a set $V = \{0, 1, 2...n\}$ of $n$ customers, including the depot $\{0\}$, and a set $A = \{(i, j)|i, j \in V, i \neq j\}$ of arcs. The set of travel costs associated with every arc is modeled as a set of $p$ discrete scenarios $S = \{1, 2, ...p\}$, where a scenario $k \in S$ is defined as an assignment of a cost value $c_{ij}^k > 0$, $\forall(i, j) \in A$. Whenever a scenario $k \in S$ is considered, the cost value for all the arcs are set to the scenario $k$. Thus, arc costs are asymmetric. In addition, a demand $d_i$ is associated with each customer $i \in V$ and it cannot be splitted in several vehicles. The fleet of vehicles $F = \{1, 2, ..m\}$ is homogeneous, and is located at the depot. Therefore, the RVRP aims at defining a set of routes which starts and ends at the depot, visits each customer once; and minimizes the maximum total cost considering all the scenarios.

Some works in the literature deal with the RVRP. A survey which outlines the RVRP with uncertainty related to demands, travel times and cost coefficients can be found in [1]. One of the most studied RVRP version involves uncertainty associated with the demands. For this purpose, authors in [2] propose a Branch-and-Bound considering the *min-max* optimization criterion; the work [3] introduces a Particle Swarm Optimization coupled with a local search; and the study [4] proposes a mathematical formulation and a Branch-and-Cut method. Closely related problems, as the Open Vehicle Routing Problem with uncertain demands has also been investigated in the literature [5]. Strategies to solve the RVRP with uncertain data on the time windows are found in [6, 7]. As far as we know, there are few works addressing the RVRP with uncertain data associated with traveling cost/time, in particular, by means of robust optimization using discrete scenarios. The following contributions are given in this study: $(i)$ we propose heuristics and a Genetic Algorithm (GA) for the RVRP which take into account two main issues, the scenarios and the asymmetric arc costs in $G$. These characteristics rely on more complex operations since a solution can have different evaluations according to the scenarios, while asymmetric costs raise the combinatorial choices to build solutions; $(ii)$ we handle uncertain data as a bounded set of discrete scenarios on a directed network.

## 2 Solution approach and experiments

An adaptation of a classical GA [8] is proposed to determine a set of routes that minimize the worst cost over all scenarios. Half of the initial population is generated by an adaptation of the best insertion heuristic and the remaining solutions are randomly generated. At each iteration of the GA, a lexicographic *min-max* strategy is applied to compare solutions, *i.e.* the costs of a solution (one per scenario) is sorted in decreasing order according to the worst case minimization. Then, a solution is said to be better than another solution if it is lexicographically smaller. The binary tournament selection mechanism is used to select the parents for applying the Order Crossover(OX) [9], and produce new solutions. Mutation inter-routes and intra-routes is also considered according to a fixed probability. If no improvement is found after a number of iteration, a partial renewal is applied to diminish the risk of premature convergence.

The computational experiments were performed on a Dell Precision 6600M, Intel Core i7-2720QM, 2.2 GHz with 16GB of RAM. The proposed GA was implemented in $C$++ on Visual Studio Express 2013, and the GLPK (GNU Linear Programming Kit), under default parameters, was used to compute optimal solutions for small size instances. Two benchmark of instances were generated to test the performance of the proposed GA for the RVRP, and to analyze the impact of the discrete scenarios. The first set contains 18 small size instances and the second one has 24 medium size instances. The small size instances consider 10, 15, and 20 customers; 2 or 3 vehicles; 10, 20 and 30 scenarios. The demand per client and the travel cost were randomly generated in [1, 50]. The vehicle capacity is selected to ensure a slack of $0.2Q$ to $0.8Q$ between the total demand $d$ and the fleet capacity $mQ$. The set of medium size instances consider a size of 50, 100 customers; 5 or 10 vehicles; 10, 20 scenarios. Each node $i$ has its coordinates randomly selected in $[0, 1000]$. The arc costs are travel times proportional to the Euclidean distance $e_{ij}$, using $[e_{ij}, (1 + d/100) \times e_{ij}]$, where $d \in \{10, 50, 100\}$ is the maximum deviation (%) to the travel time. The vehicle capacity is $Q = 1000$, while the demands are random integers smaller than $0.9\times$ $mQ$.

The results indicate that GLPK was not able to prove optimality in up to 4 hours for instances with 15 customers and 30 scenarios, while the proposed GA found good solutions within 8 seconds in a small average gap of 3.14%. Moreover, the proposed GA found 7 optima out of 10 optima (found by GLPK) for the small size instances. For the set of medium size instances, initially the GA parameters were fine-tuned. Then, five runs have been performed per instance using different seeds. Considering the five runs, the minimum cost (Minc), the average cost (Avgc), and the average time (Avgt) were used as performance metrics. Results indicate that the GA consumes 118.4 and 573.21 seconds for instances with $n = 50$, and $n = 100$, respectively. Moreover, when $n$ and $p$ increase, the complexity of the problem and the running time raise. Table 1 presents a sample of the results obtained over the small and medium size instances. Results demonstrate that the RVRP with discrete scenarios is a very hard problem to solve. Conerning future works, different heuristics and metaheuristics will be proposed, and applied for large instances. We have been working on a rich RVRP version with other uncertain parameters, solved by means of robust optimization.

| Small Size Instances | GA | | GLPK | |
|---|---|---|---|---|
| | Gap | T(s) | Gap | T(s) |
| n10-m2-p10 | 0.00 | 2.7 | 0.00 | 8.2 |
| n10-m3-p30 | 0.00 | 4.1 | 0.00 | 38.9 |
| n15-m3-p30 | 2.35 | 6.2 | 1.64 | 14400.0 |
| n20-m2-p20 | 5.87 | 4.1 | 2.17 | 14400.0 |

| Medium Size Instances | GA | | |
|---|---|---|---|
| | Minc | Avgc | Avgt |
| n50-m5-p10-d10 | 9981 | 10316 | 106.5 |
| n50-m10-p20-d50 | 16042 | 16822 | 124.1 |
| n100-m10-p10-d10 | 16344 | 17039 | 572.8 |
| n100-m20-p20-d100 | 34953 | 37320 | 443.3 |

**Table 1.** Sample of results for small and medium size instances

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