Parameter selection in VNS for the k-labelled spanning forest problem

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1 Introduction

A graph whose edges are labelled represents a Multimodal Transportation Networks where each label denotes a different company managing that link. In multimodal transportation it is desirable to guarantee a complete connection between the terminal nodes of the network by using the minimum number of provider companies [1]. The purpose is to minimize the cost and the overall complexity of the network. In real-world applications it is interesting to optimize this factor with an upper bounds on the number of transportation providers available [2]. This is the k-labelled spanning forest (kLSF) problem that has been shown NP-hard [3] and therefore any practical solution approach requires heuristics.

The kLSF problem can be formalized as follows. Let $G = (V, E, L)$ be a labelled, undirected graph, where V is the set of vertices or nodes, E the set of edges or links, that are labelled on the set L of labels, and let $\bar{k}$ be a positive integer. The problem consists in finding a subgraph S of G with no more than $\bar{k}$ labels minimizing the number of connected components of S. The kLSF problem is a generalization of the MLST problem which consists in finding the spanning tree of the graph with the minimum number of labels [4]. Moreover, a solution to the MLST problem is a solution also to the kLSF problem if the solution tree has not more than $\bar{k}$ labels. Therefore solution approaches to the kLSF problem can be based in MLST heuristics. Several metaheuristics have been applied to the MLST problem in [5, 7, 6, 8–11]. In this paper we consider the application of VNS metaheuristic for the kLSF and the selection of the values for its parameters in order to get the best performance.

2 VNS for the kLSF problem

The key idea of Variable Neighbourhood Search (VNS) [12] is to define a neighbourhood structure for the solution space, and to explore different increasingly distant neighbourhoods whenever a local optimum is reached. Our VNS implementation for the kLSF problem is motivated by the successful VNS proposed for the MLST problem in [10]. Consider the following notation. Each solution S of the kLSF is encoded by a binary string $S = (l_1, l_2, ..., l_n)$, where $l_i = 1$ if label i is included in the solution S, and $l_i = 1$ otherwise. Let $\rho(S_1, S_2) = |S_1 - S_2|$ be the Hamming distance between any two solutions S_1 and S_2. Denoting the qth neighbourhood of a solution S by $N_q(S) = \{S' \subseteq L : \rho(S, S') = q\}$, a set of q_{max} neighbourhoods (N_q, with $q = 1, 2, ..., q_{max}$) is selected, where the parameter q_{max} is the maximum size of the set of neighbourhood structures. The value of q_{max} represents an important parameter to tune in VNS in order to obtain an optimal balance between intensification and diversification capabilities [12].

VNS starts with an initial solution S obtained by adding labels at random from scratch until $\bar{k}$ labels are selected. Then the algorithm applies the shaking phase to S. The shake consists in selecting at random of a solution S' from the neighbourhood $N_q(S)$ of the current solution S. The parameter q varies from 1 to q_{max} throughout the execution. In order to construct $N_q(S)$, the algorithm starts by deleting a random labels from S, and then including at random further labels from the unused set $(L - S)$, if a further expansion of the neighbourhood structure is required (case $q > |S|$).

Then a local search is applied to the incumbent solution S'. The local search tries to drop each of the labels in S', and then to add further labels following a greedy rule, until $\bar{k}$ are reached. After the local search, if an improvement has not been obtained, the neighbourhood is increased ($q \leftarrow q+1$). The process of changing the neighbourhood structure when the local search is trapped at a local minimum represents the core idea of VNS and provides an increased progressive diversification. Otherwise, if an improvement is obtained, the algorithm moves to the improved solution ($S \leftarrow S'$) and restarts the shaking with the smallest neighbourhood ($q \leftarrow 1$). This procedure is repeated until the predetermined stopping conditions are reached, providing the best solution S.

3 Experimental results

We run the experiments to analyse the performance with different values for q_{max} on 8 different datasets having numbers of vertices $|V| \in \{100, 200\}$, number of labels $|L| = \alpha|V|$ for $alpha \in \{0.25, 0.5, 1, 1.25\}$, and edges $|E| = |V|(|V| - 1)$. For each combination of $|V|$ and $|L|$, 10 different problem instances were considered, for a total of 80 instances. Following [3], the parameter $\bar{k}$ was determined as $\lfloor |V|/2^j \rfloor$, where j is the smallest value such that the generated instances did not report a single connected solution when solved with the MVCA heuristic for the MLSTP. The Maximum Vertex Covering Algorithm (MVCA) is a polynomial time heuristic for the MLST problem proposed in [4] and successively improved in [13]. For each dataset, solution quality is evaluated as the average number of connected components among the 10 problem instances. A maximum allowed CPU time was chosen as stopping condition for all the parameter setting, determined experimentally with respect to the dimension of the considered datasets. The objective of the experiments is to test three different strategies for selecting q_{max}. These strategies consist of giving to q_{max} a fix value for for all the instances, a value proportional to the label set size $|L|$ and a value proportional to the solution size $|S|$. The experiments show the superiority of applying the rule $q_{max} = 4/3|S|$ used in [10] for the MLST problem.

References

1. Van-Nes, R.: Design of multimodal transport networks: A hierarchical approach. Delft University Press (2002).
2. Miller, J. S.: Multimodal statewide transportation planning: A survey of state practices. Transportation Research Board, National Research Council, Virginia, US, (2006).
3. Cerulli, R., Fink, A., Gentili, M., Raiconi, A.: The k-labeled spanning forest problem. Procedia - Social and Behavioral Sciences 108 (2014), pp. 153-163.
4. Chang, R. S., Leu, S. J.: The minimum labelling spanning trees. Information Processing Letters 63 (1997), pp. 277-282.
5. Brüggemann, T., Monnot J., Woeginger, G. J.: Local search for the minimum label spanning tree problem with bounded colour classes. Operations Research Letters 31 (2003), pp. 195-201.
6. Xiong, Y., Golden, B., Wasil, E.: Improved heuristics for the minimum labelling spanning tree problem. IEEE Transactions on Evolutionary Computation 10 (2006), pp. 700-703.
7. Cerulli, R., Fink, A., Gentili, M., Voss, S.: Metaheuristics comparison for the minimum labelling spanning tree problem. In: B. L. Golden, S. Raghavan and E. A. Wasil, editors, The Next Wave on Computing, Optimization, and Decision Technologies, Springer-Verlag, New York, 2005 pp. 93-106.
8. Consoli, S., Moreno-Pérez, J. A., Darby-Dowman, K., Mladenović, N.: Discrete particle swarm optimization for the minimum labelling Steiner tree problem. In: N. Krasnogor, G. Nicosia, M. Pavone and D. Pelta, editors, Nature Inspired Cooperative Strategies for Optimization, Studies in Computational Intelligence 129, Springer-Verlag, New York, 2008 pp. 313-322.
9. Consoli, S., Darby-Dowman, K., Mladenović, N., Moreno-Pérez, J. A.: Greedy randomized adaptive search and variable neighbourhood search for the minimum labelling spanning tree problem. European Journal of Operational Research 196 (2009), pp. 440-449.
10. Consoli, S., Darby-Dowman, K., Mladenović, N., Moreno-Pérez, J. A.: Variable neighbourhood search for the minimum labelling Steiner tree problem. Annals of Operations Research, 172 (1) (2009) pp. 71–96.
11. Consoli, S., Moreno-Pérez, J. A.: Solving the minimum labelling spanning tree problem using hybrid local search. Electronic Notes in Discrete Mathematics 39 (2012), pp. 75-82.
12. Hansen, P., Mladenović, N., Brimberg, J., Moreno-Pérez, J.A.: Variable Neighborhood Search. In G. Mitchel, J.-Y. Potvin, editors, Handbook of Metaheuristics. pp. 61–86. Springer, 2010.
13. Krumke, S. O., Wirth, H. C.: On the minimum label spanning tree problem. Information Processing Letters 66 (1998), pp. 81–85.